

# Impact & Crashworthiness Laboratory

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by

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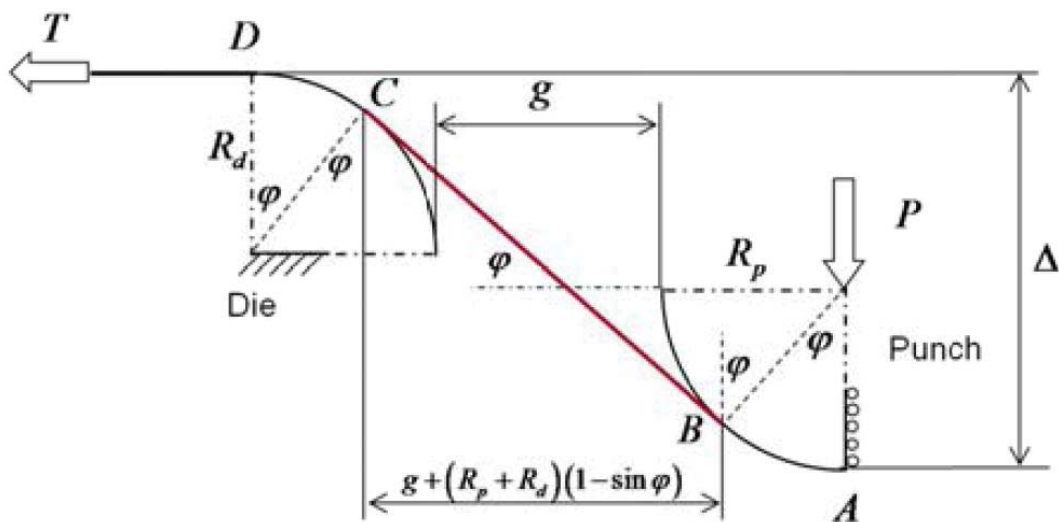
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# Analytical Solution on the Failure of Strips under Bending and Tension

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## Abstract

The analysis of stretch-bending of a strip is important in understanding failure initiation in the sheet metal forming process. Most of the current work in this area has been purely experimental or numerical simulation. In the paper, a procedure for constructing an analytical solution for failure of strips under bending and tension was developed. One important aspect of the derivation was the consideration of the logarithmic strain instead of engineer strain, as was done in the existing solutions found in the literatures. The other important aspect is the assumption of tension/bending damage coupling. The input data of the analytical solution are the geometry of the tool (die radius), sheet thickness, parameters of the plasticity hardening rule, magnitudes of fracture strains under uni-axial tension and plane strain, and the tension/bending damage coupling coefficient. The present closed-form solution was validated by test results of the present authors as well as data from the open literature. A good correlation is obtained between the measured and predicted radius to thickness ratio ( $R/t$ ) to fracture under a given pre-tension. The results of the paper will be useful in the design of stamping and deep drawing operation.

**Keywords:** Ductile fracture, bending, tension, analytical solution, forming

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